

$R_a$  = Armature resist.  
 $L_a$  = Armature induct.  
 $J$  = Load inertia  
 $B$  = friction constant  
 $i_a(t)$  = armature current  
 $V_a(t)$  = armature voltage  
 $\omega$  = rotation speed  
 $\theta$  = angular displacement  
 $T$  = motor torque  
 $k_t$  = torque constant  
 $k_b$  = back emf constant  
 $E_b$  = Back emf voltage

KVL  $\Rightarrow$

$$V_a(t) - E_b(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} \quad (1)$$

$$E_b(t) = k_b \omega(t) \quad (2)$$

$$J \omega'(t) + B \omega(t) = T(t) \quad (3)$$

$$T(t) = k_t i_a(t) \quad (4)$$

$$\omega(t) = \theta'(t) \quad (5)$$

S-domain

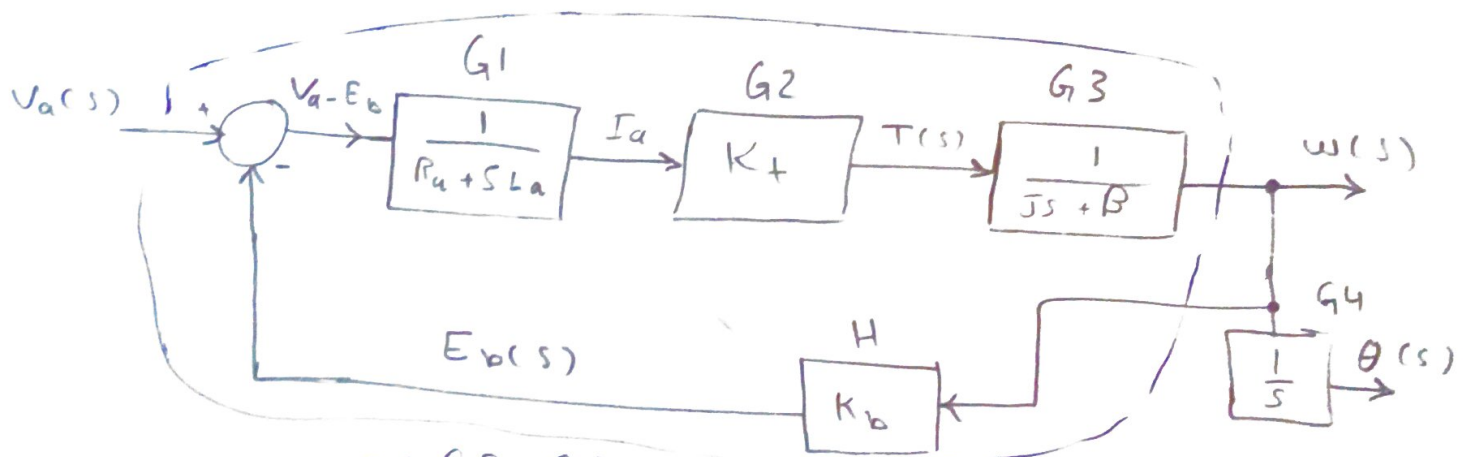
$$V_a(s) - E_b(s) = (R_a + L_a s) I_a(s) \quad (1')$$

$$E_b(s) = k_b \omega(s) \quad (2')$$

$$(sJ + B) \omega(s) = T(s) \quad (3')$$

$$T(s) = k_t I_a(s) \quad (4')$$

$$\omega(s) = s \theta(s) \quad (5')$$



$$\frac{\omega(s)}{V_a(s)} = \frac{G_1 G_2 G_3}{1 + H G_1 G_2 G_3}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{G_1 G_2 G_3}{1 + H G_1 G_2 G_3} * G_4 \quad (1)$$

$$G_1 = \frac{1}{L_a s + R_a}, \quad G_3 = \frac{1}{J s + B}$$

$$G_2 = K_t, \quad G_4 = \frac{1}{s}$$

$$1-1 = K_b$$

$$G_1 G_2 G_3 = \frac{K_t}{(L_a s + R_a)(J s + B)}$$

TF for ω(s) output

$$\frac{\omega(s)}{V_a(s)} = \frac{K_t}{(L_a s + R_a)(J s + B) + K_t K_b}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{\omega(s)}{V_a(s)} \times \frac{1}{s} = \frac{K_t}{s(L_a s + R_a)(J s + B) + s K_t K_b}$$

$$R_a = 1 \Omega, \quad L_a = 1 \text{ mH} \xrightarrow{K_t}, \quad K_m = 10 \xrightarrow{N \cdot m / A}, \quad K_b = 0.1 \text{ V s / m}$$

$$B = 0.5 \xrightarrow{N / s}, \quad J = 2 (K_g \cdot m)$$

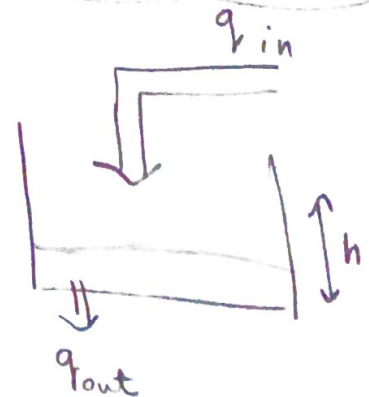
Task  $\Rightarrow$  input T load change B. Diagram

follow rate  $q = m^3/s$

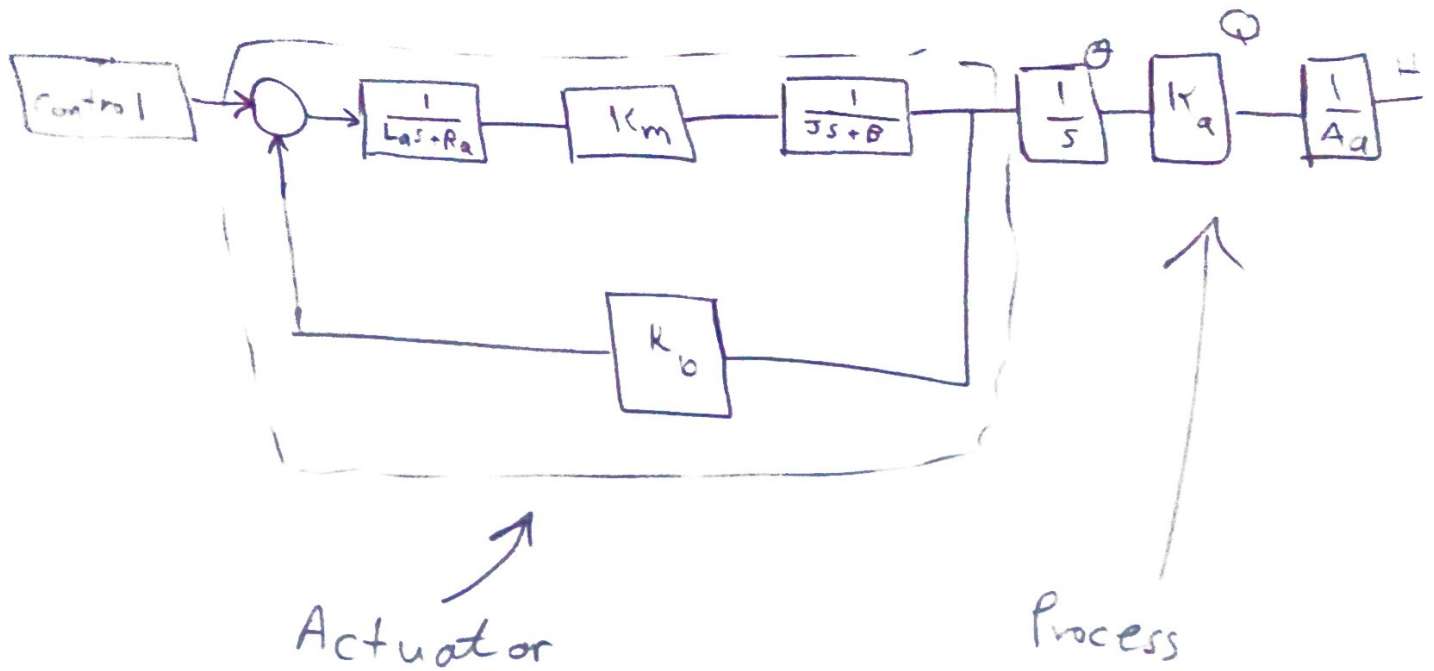
$$q_{in} = \frac{dh(t)}{dt} \times A$$

$$q_{in}(s) = s H(s) \times A$$

$$\frac{H(s)}{q_{in}(s)} = \frac{1}{A s}$$



(2)



Task  $\Rightarrow$  add  $T_{read}$  to the motor  
and resketch the block diagram

Hint:  $\theta$  تدرج ال  $T_{read}$  بتغير القياس في الوتور